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Construction of Airplane Landing Mathematical Models With Drag Chute

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Abstract

In this article, we analyze the dynamics of a airplane landing using a parachute called a drag chute. As long as the aircraft makes a landing until it comes to a complete stop, the airplane experiences two times the air drag, namely the air resistance that appears before the drag chute is used and after it is used. From the calculation, we obtain that the distance is shorter than without using a drag chute.

***Keywords**: air drag; drag chute; mathematical models.*

Introduction

Usually, a parachute is used to reduce the speed of an object. For example, parachutists use parachutes to reduce their speed during free fall so that the possibility of an accident is smaller. The dynamics associated with this parachute has also been studied [1]–[3]. Currently, parachutes are also used to reduce the speed of airplane when landing, even the ability of aircraft to make short landings is one of the criteria for selecting a medium-type transport airplane [4]–[6].

The dynamics related to the motion of an aircraft using a drag chute cannot be separated from the role of air drag. In most cases considered in physics, air resistance is usually neglected. However, the effect of air resistance sometimes needs to be taken into account so that the analytical results obtained are more real. This is because every object that moves around the surface of the earth will always experience air resistance. Therefore, it is important to involve air resistance in studying the dynamics of moving objects.

The effect of air resistance is to reduce the speed of a moving object. Generally, the amount of air resistance is directly proportional to the speed of the object. The faster the object moves, the greater the air resistance [7]. Studies on the effect of air resistance on object motion have been carried out both for simple objects such as projectile motion [8] and free falling objects [9]–[13] as well as objects on a large scale [14], [15].

In this article, we analyze the motion dynamics of an airplane that uses a drag chute to shorten the landing distance. This article is structured as follows: First, we describe the airplane motion, second, we find a solution to the equations of motion and finally, we analyze the results.

Methods

The analysis is carried out when the airplane has touched the runway (touchdown) and is decelerated to a complete stop. We assume that the air resistance is proportional to the speed of the airplane. Mathematically, the air drag is expressed as

$$F_D = -bv \tag{1}$$

where b is the proportionality constant and v is the velocity of airplane. The negative sign in equation 1 indicates that the air drag has the opposite direction to the motion of airplane. In addition, we examine the effect of air drag on the airplane in two steps, namely before and after the drag chute is used. Before the drag chute is used, we label the proportionality constant with b_1 and after the drag chute is used we label it with b_2. Thus the drag force experienced by the airplane is given by

$$F_D = \begin{cases} -b_1 v, & 0 \leq t < T \\ -b_2 v, & t \geq T. \end{cases} \tag{2}$$

where T is the time when the drag chute starts to be used by the airplane. Also, we ignore the time it takes for the drag chute to open completely.

Result and Discussion

In general, the equation of motion of the airplane using drag chute is given by

$$m\frac{dv}{dt} = -bv \tag{3}$$

where m is the mass of the airplane, v is the velocity of the airplane and b is the proportionality constant. Equation 1 will be used to find the velocity and position of the airplane for a certain time both before and after drag chute is used.

Before Drag Chute is Used

Before the drag chute is used, the proportionality constant of air drag is b_1 and the time required in this condition is $0 \leq t < T$. The equation of motion in this state is

$$m\frac{dv}{dt} = -b_1 v. \tag{3}$$

Using the integral technique, the velocity and distance traveled by the airplane in this state are given by

$$v(t) = v_0 e^{-\frac{b_1}{m}t} \tag{4}$$

$$x(t) = \frac{mv_0}{b_1}\left[1 - e^{-\frac{b_1}{m}t}\right]. \tag{5}$$

To obtain equations 5 and 6, we use the initial conditions. When $t = 0$, the velocity and position of the airplane are $v = v_0$ and $x = 0$, respectively. Note that when $t \to \infty$ the velocity of the airplane goes to zero and the position of the airplane is $\frac{mv_0}{b_1}$. However, the velocity and distance covered by the airplane is limited to $t = T$ because after that time the drag chute will be opened.

After Drag Chute is Used

In this situation the initial velocity of the airplane is not v_0. The initial velocity of airplane in this condition is the same as the velocity when the drag chute starts to be used or $v = v(t = T)$. The equation of motion at this state is given by

$$m\frac{dv}{dt} = -b_2 v. \tag{6}$$

In the same way as in the unopened drag chute state, the airplane speed is given by

$$v(t) = Ce^{-\frac{b_2}{m}t}. \tag{7}$$

where C is a constant. Note that the velocity of the airplane before and after the drag chute is opened is the same. Under these conditions we can obtain the constant C in equation 8, namely

$$C = v_0 e^{-\frac{1}{m}(b_1 - b_2)\,T}. \tag{8}$$

Substituting equation 9 into equation 8, the velocity of airplane at time $t \geq T$ is given by

$$v(t) = v_0 e^{-\frac{1}{m}[(b_1 - b_2)T + b_2 t]}. \tag{10}$$

Then by doing the integral in equation 10, the distance traveled by the airplane at time $t \geq T$ is

$$x(t) = x_0 + \frac{mv_0}{b_2} e^{-\frac{b_1}{m}T}\left(1 - e^{-\frac{b_2}{m}(t-T)}\right) \tag{11}$$

where x_0 is the distance traveled by the airplane while the drag chute has not been opened, namely equation 6. Thus the total distance traveled by the aircraft before and after the drag chute is opened is

$$x(t) = \frac{mv_0}{b_1}\left\{1 - \left[1 - \frac{b_1}{b_2}\left(1 - e^{-\frac{b_2}{m}(t-T)}\right)\right] e^{-\frac{b_1}{m}T}\right\}. \tag{9}$$

Equation 12 is valid for $t \geq T$. In general, we can use equation 12 for $t \geq 0$. If the travel time of the airplane while the drag chute has not been opened is T and after it is opened is t, the total time required during landing is

$$t' = T + t. \tag{13}$$

Using equation 13, the distance traveled by the airplane becomes

$$x = \frac{mv_0}{b_1}\left\{1 - \left[1 - \frac{b_1}{b_2}\left(1 - e^{-\frac{b_2}{m}(t-2T)}\right)\right] e^{-\frac{b_1}{m}T}\right\}. \tag{14}$$

In equation 14 we have changed the variable t' back to t. Equation 14 applies to $t \geq 0$.

Discussion

From equation 5 and 10, the speed of the plane while on the runway is

$$v(t) = \begin{cases} v_0 e^{-\frac{b_1}{m}t}, & 0 \leq t < T \\ v_0 e^{-\frac{1}{m}[(b_1-b_2)T+b_2 t]}, & t \geq T \end{cases} \tag{10}$$

and the distance traveled during landing is given by equation 14. To see the effect of using a drag chute in shortening the landing distance, we compare equations 14 and 6. For a long time, namely $t \to \infty$, we obtain that the comparison is

$$x_a = \left[1 - \left(1 - \frac{b_1}{b_2}\right) e^{-\frac{b_1}{m}T}\right] x_b \tag{14}$$

where x_a and x_b are the distance traveled by the airplane before and after the drag chute is used, respectively. For $b_2 \gg b_1$, we get that $x_a < x_b$. Then the condition $b_2 > b_1$ also gives the same result, namely $x_a < x_b$. This shows that the use of drag chute on the airplane will reduce the stopping distance.

Conclusion

The analysis shows that the use of drag chute can shorten the stopping distance of the airplane. By assuming that the air drag is only directly proportional to the speed and the constant of proportionality, the stopping distance of the aircraft becomes shorter. However, the assumptions used are still relatively simple. Quantities such as temperature, air density and cross-sectional area of the drag chute will add to the accuracy of determining the distance traveled by the airplane during landing. In the future, research involving this quantity can be reviewed again.

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