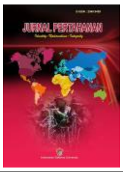

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Stability Analysis of Mathematical Models of Corrupt Behavior with Death Penalty Sanction

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Abstract

Indonesia is one of the countries with a fairly solid economy. Indonesia's foreign exchange reserves reached US$146,383.75 million. This amount should help improve the welfare of its people. However, the fact is that corruption harms the state budget so that people's welfare cannot be fulfilled. As of 2023, Indonesia has lost at least Rp28.412.786.978.089 in 791 cases involving 1,695 suspects. Even after serving their prison sentences, these corruptors' actions fail to provide a significant deterrent effect. Therefore, the imposition of the death penalty is necessary to deter corruption crimes in Indonesia. In this study, we will analyze the stability of the critical point in the mathematical model that incorporates the modified imposition of the death penalty on the corruptor population (C). In the mathematical model of corruption, there are three populations: Susceptible population (S), Corruptor population (C), and Honest population (H). The goal is none other than to minimize and eliminate the corrupt population. The results indicated that there were two critical point conditions: (S1, C1, H1) and (S*, C*, H*). The critical points (S*, C*, H*) show that the corruptor population and honest population still exist. As for the critical point (S1, C1, H1), it appears that the corruptor population and honest population are extinct. This study shows that the imposition of the death penalty in corruption cases can exterminate the corruptor population and certainly has implications for the extinction of the honest population. The value of the benchmark parameter clearly influences this result. Therefore, it can be concluded that applying this research may reduce corruption cases and enhance defense management by promoting accountable state financial management.

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INTRODUCTION

Indonesia is one of the countries in Southeast Asia with an economy ranked 16th in the world as measured by GDP (Australian Government, 2023). In 2023, Indonesia's total GDP per capita was $4,876.3, with a GDP growth rate of 5% per year (Group, 2023). Given Indonesia's relatively favorable economic conditions, it is crucial to apply income equality effectively. Some policies to overcome income distribution inequality in Indonesia include cash observations that suggest that this policy solely targets the community. subsidies, price subsidies, goods subsidies, progressive taxation policies, provision of employment, and credit insurance for MSMEs (Ratna et al., n.d.). If observed, this policy is only focused on the community. On the other hand, corruption problems in Indonesia are common and can harm the country's economy.

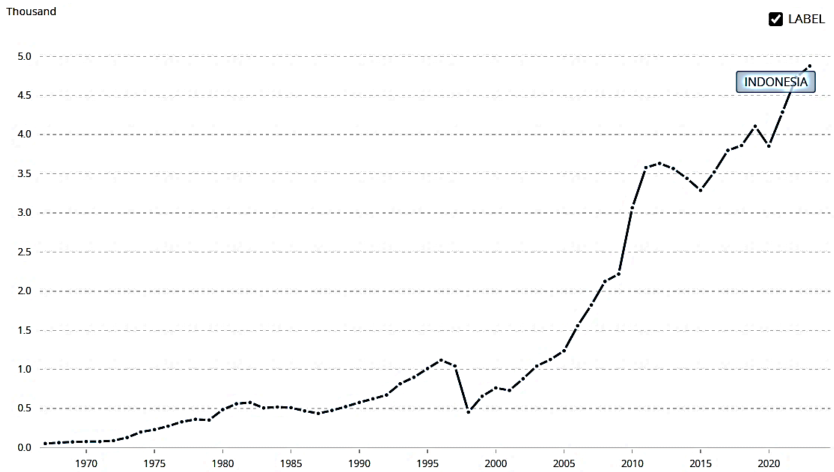

Figure 1. GDP per Capita of Indonesia
Source: (World Bank Group, 2023)

According to etymology, corruption can be interpreted as a situation that was originally intact, desirable, beneficial, and right, and there were no problems but became the opposite due to undesirable actions through fraud, bribery, forgery, and so on. The perpetrators of this act of corruption are referred to as corruptors (Yani, 2022). Corruption can have a massive impact on the country, including social consequences such as increased poverty, effects on government bureaucracy, influences on politics and democracy, repercussions for law enforcement, and implications for defense and security (Syauket & Wijanarko, 2024). In Indonesia, the government agency that deals with corruption issues is the Corruption Eradication Commission (KPK), where every implementation of its duties and authorities is based on legal certainty, openness, accountability, public interest, proportionality, and respect for human rights (Kemenkumham, 2019). On a scale of 0-100, Indonesia scored 34 as the country with the highest level of corruption in the world (Transparency International, 2024). Based on KPK data, there are at least 8 cases of violation of KPK ethics, four lists of fugitives, and 14 lists of unresolved case arrears until 2023 (Nicola & Rohman, 2023). The corruption problem can be modeled in the form of a mathematical model. The purpose of mathematical modeling is to find optimal solutions to phenomena that occur in the real world (Aprilia & Panjaitan, 2022).

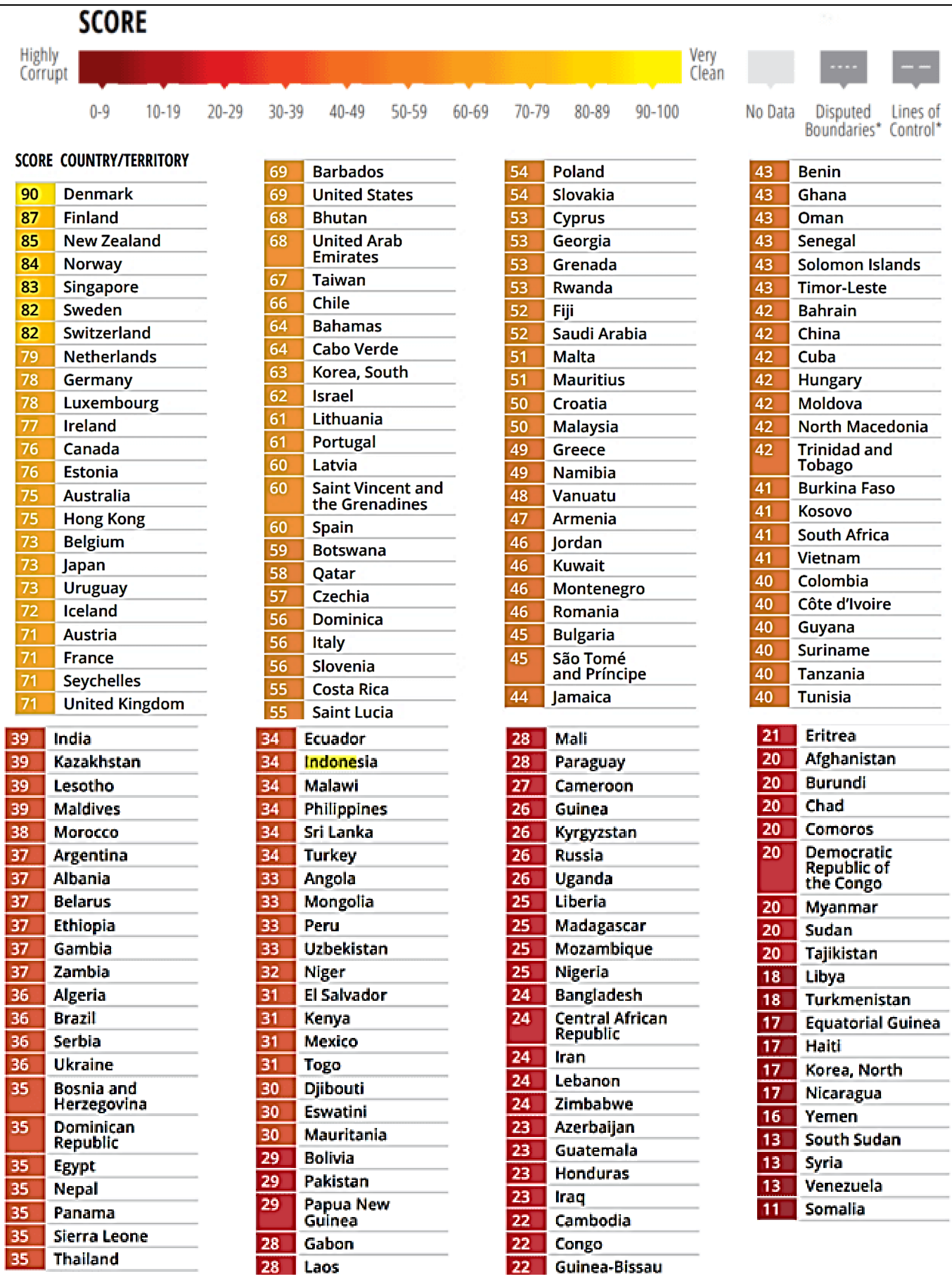

Figure 2. The Perceived Levels of Public Sector Corruption in 180 Countries
Source: (Transparency International, 2024)

Corruption is one of the actions that can harm the country's economy. In Indonesia, anyone who commits a corruption crime will receive a minimum sentence of 2 years and a maximum sentence of 20 years (President RI, 2023). This rule is only limited to the period of serving the sentence. Meanwhile, if the losses suffered by the state are extraordinary, then the corrupt perpetrator still has the opportunity to be free after serving a sentence. From the perspective of Human Rights (HAM), the death penalty cannot be carried out because it contradicts Pasal 28I UUD 1945 (Cahyani et al., 2023). However, if this is left unchecked, the country will experience tremendous destruction due to corruption. Although the death penalty is considered something that violates human rights, some countries still administer the death penalty to prisoners depending on the type of crime committed (Amnesty International, 2024).

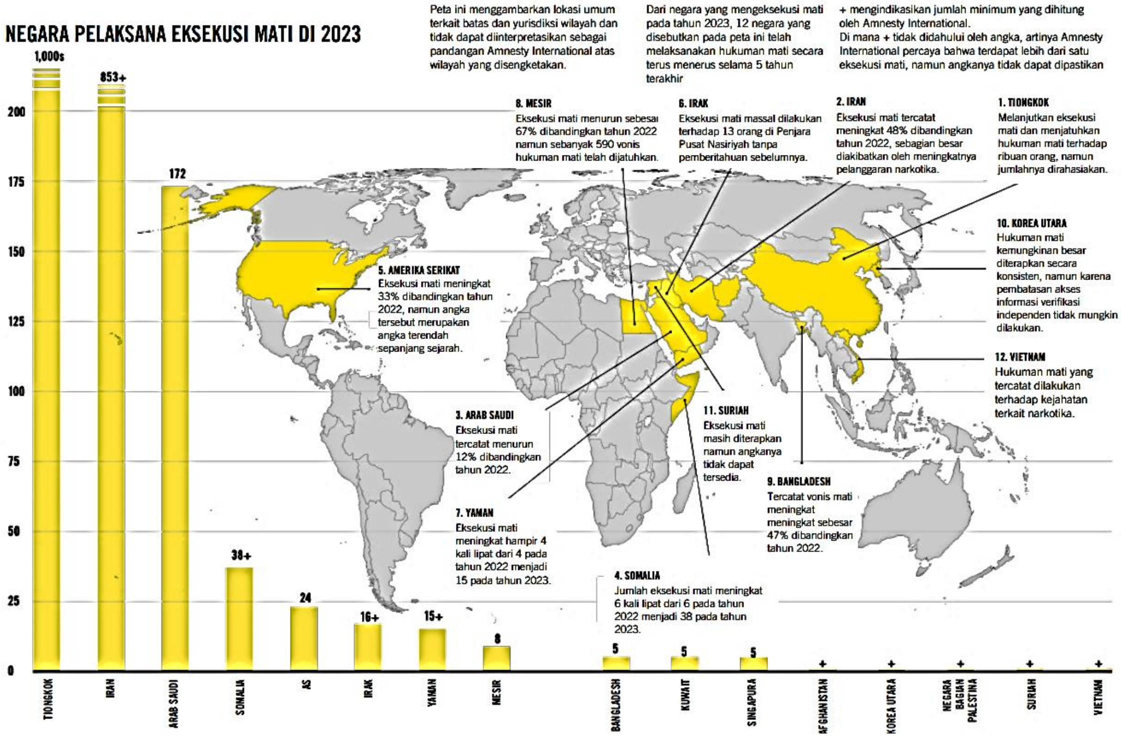

Figure 3. Countries Implementing the Death Penalty
Source: (Amnesty International, 2024)

MATERIALS

Old Mathematic Model

$$\frac{dS}{dt} = \Pi - \kappa CS + \omega H - \mu S$$
$$\frac{dC}{dt} = \kappa CS - \beta C - \mu C \qquad (1)$$
$$\frac{dH}{dt} = \beta C - \omega H - \mu H$$

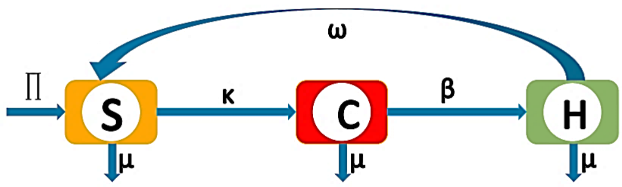

Figure 4. Corruption Dynamic Model
Source: (Ahmed et al., 2025)

The model proposed by Ahmed et al., provides an explanation where $t > 0$; $S(0) \geq 0$; $C(0) \geq 0$; $H(0) \geq 0$ and definition $S(t) :=$ Total susceptible population at the time t; $C(t) :=$ Total corrupt population at the time t; $H(t) :=$ Total honest population at the time t; $\Pi :=$ Number of individuals in the population; $\kappa :=$ Rate of spread of corruption from the population S to C; $\omega :=$ Rate of conversion to population H; $\mu :=$ Rate of natural death; and $\beta :=$ Rate of transition of population C to H. The model explains that:

1. The S Population increases through the rate of individuals entering the S population (Π) and the conversion rate to the H population (ωH). D On the other hand, population S decreases due to the rate of spread of corruption in the population S into C ($-\kappa CS$) and the rate of natural mortality ($-\mu S$).
2. The C Population increases due to the rate of spread of corruption in the S to C population (κCS). Se While the decrease in this population is due to the transition rate of the C population into H ($-\beta C$) and the natural death rate ($-\mu C$).
3. The H Population increases due to the transition rate of the C to H population (βC). In addition, this population decreases due to the conversion rate to the H population ($-\omega H$) and the natural mortality rate ($-\mu H$).

Proposed Mathematic Model

In corruption cases that occur, some corruptors who have served their sentences and become a population H can become population C and get caught in corruption cases again. This is because the corruptor did not repent after being released from punishment. For this reason, it is necessary to have criminal sanctions in the form of the death penalty, which can have a deterrent effect on corruptors from the perspective of criminal law, criminology, and victimology (Munasto, 2022). Based on the old model and the concept of the death penalty sanction, the researcher proposes a mathematical model of corruption behavior with the death penalty sanction with an additional parameter θ ≔ The number of individuals serving the death penalty sanction and γ ≔ The rate of reverse transition of population C to H.

$$\begin{aligned}\frac{dS}{dt} &= \Pi - \kappa CS + \omega H - \mu S\\ \frac{dC}{dt} &= \kappa CS - \beta C + \gamma H - \mu C - \theta C\\ \frac{dH}{dt} &= \beta C - \omega H - \gamma H - \mu H\end{aligned} \tag{2}$$

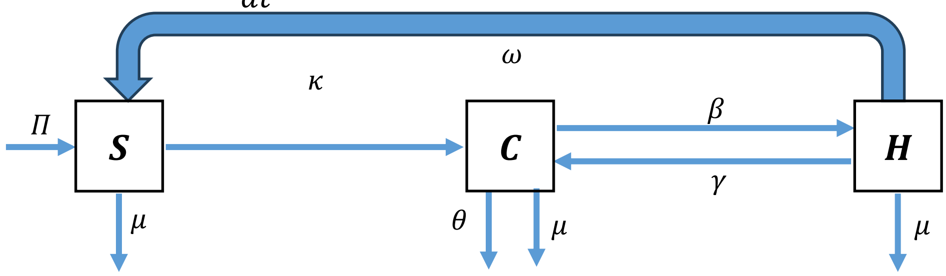

Figure 5. Corruption Dynamics Model with Death Penalty Sanctions
Source: Researcher (2025)

METHODS

Differential Equation System

A differential equation is a form of equation that contains derivatives with 1 or more dependent variables against 1 or more independent variables of a function (Nuryadi, 2018). In general, an nth-order differential equation can be written as

$$y^{(n)}(x) = f\left(x, y, y', y'', \dots, y^{(n-1)}\right) \tag{3}$$

A differential equation can be said to be linear if it satisfies.

$$a_n y^n + a_{n-1} y^{n-1} + \cdots + a_1 y' + a_0 y = b, \forall a_n \neq 0 \tag{4}$$

For $b = 0$ It is said to be homogeneous, while $b \neq 0$ Is said to be non-homogeneous. Homogeneous differential equations have solutions in characteristic equations.

$$a_n k^n + a_{n-1} k^{n-1} + \cdots + a_1 k + a_0 = 0 \tag{5}$$

Where the solution of the characteristic equation satisfies one of the conditions:

1. If $k_1 \neq k_2 \neq \ \ldots \neq k_n, \forall k_n \in \mathbb{R}$ then the solution is
$$y(x) = c_1 e^{k_1 x} + c_2 e^{k_2 x} + \cdots + c_n e^{k_n x} \tag{6}$$
2. If $k_1 = k_2 = \ \ldots = k_n, \forall k_n \in \mathbb{R}$ then the solution is
$$y(x) = c_1 e^{mx} + c_2 x e^{mx} + c_3 x^2 e^{mx} \ldots + c_n x^{n-1} e^{mx} \tag{7}$$
3. If n is even and $k_{12} = a_1 \pm b_1 \neq k_{34} = a_2 \pm b_2, \forall k_n \in \mathbb{C}$ then the solution is
$$y(x) = e^{a_1 x}(c_1 \cos b_1 x + c_2 \sin b_1 x) + e^{a_2 x}(c_3 \cos b_2 x + c_4 \sin b_2 x) + \cdots + e^{\frac{a_n x}{2}}\left(c_{n-1} \cos b_{\frac{n}{2}} x + c_n \sin b_{\frac{n}{2}} x\right) \tag{8}$$
(Firdaus, 2020).

A system of differential equations is a collection of more than one differential equation that can be organized in the form of

$$\begin{aligned} \frac{dx_1}{dt} &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + g_1 \\ \frac{dx_2}{dt} &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + g_2 \\ &\vdots \\ \frac{dx_n}{dt} &= a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + g_n \end{aligned} \tag{9}$$

The general solution of the system of nonhomogeneous differential equations is

$$X = X_h + X_p \tag{10}$$
(Ratna et al., n.d.).

where $X_h :=$ Homogeneous solution and $X_p :=$ Particular solutions searched through

1. Determine a system of differential equations of the form
$$x' = Ax + g \text{ with} \tag{11}$$
$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \text{ and } g = \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix} \tag{12}$$
where $A :=$ coefficient matrix and $g :=$ vector function (Imrona, 2002).
2. Determine the eigenvalue λ and the corresponding eigenvector x of matrix A
$$\det(A - \lambda I) = 0 \text{ and } (I + A)x = 0 \tag{13}$$
3. Constructing the nonsingular matrix P from the obtained eigenvectors
$$P = [x_1 \quad x_2] \tag{14}$$
4. Since $\det(P) \neq 0$ we can find P^{-1} which is the inverse of matrix P
5. Determine the diagonal matrix D
$$D = P^{-1}AP \tag{15}$$
6. Specifies the exponential matrix e^{At}
$$e^{At} = Pe^{Dt}P^{-1} \tag{16}$$
7. So that a homogeneous solution is obtained X_h
$$X_h = e^{At}C, \text{ with } C \text{ an arbitrary constant.} \tag{17}$$

8. Next determine the exponential matrix $e^{-A\tau}$ and function $g(\tau)$, then calculate the particular solution X_p

$$X_p = e^{At} \int_0^t e^{-A\tau} g(\tau) d\tau \text{ (Ratna et al., n.d.).} \quad (18)$$

Stability Analysis

y_C is a critical point of the differential equation $y' = f(y)$ if it satisfies $f(y_C) = 0$. The critical point will satisfy one of the conditions:

1. Stable if $f(y) > 0,\ \forall y \neq y_C$ around y_C.
2. Unstable if $f(y) < 0,\ \forall y \neq y_C$ around y_C.
3. Semistable if the critical point is in the stable and unstable region (Nagy, 2015a).

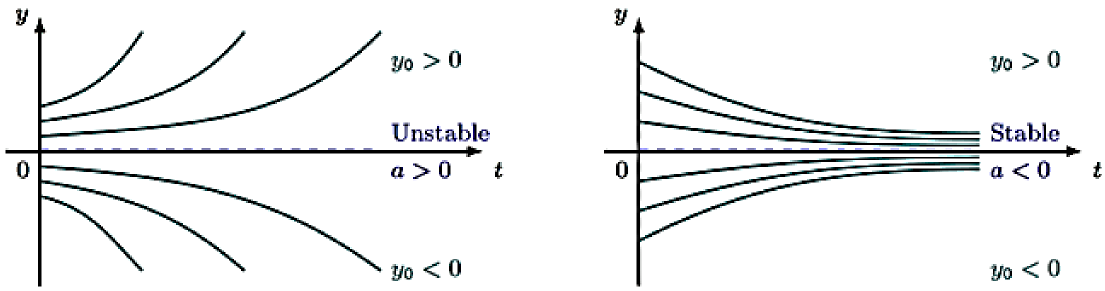

Figure 6. Critical Point Stability

Source: (Nagy, 2015)

RESULTS AND DISCUSSIONS

Results

Based on the new mathematical model, a stability analysis will be conducted on the mathematical model.

1. Critical Point

By calculating $\frac{dS}{dt} = \frac{dC}{dt} = \frac{dH}{dt} = 0$ the critical point is obtained

$$(S_1, C_1, H_1) = \left(\frac{\Pi}{\mu}, 0,0\right) \quad (19)$$

with $\frac{\Pi}{\mu} > 0$ and

$$(S^*, C^*, H^*) = \left(\frac{\mu^2+(\beta+\theta+\omega+\gamma)\mu+(\beta+\theta)\omega+\theta\gamma}{\kappa(\mu+\omega+\gamma)}, \frac{\mu^3-(\beta+\theta+\omega+\gamma)\mu^2+(\kappa\Pi-\theta\omega-\theta\gamma)\mu+\kappa\Pi\omega+\kappa\Pi\gamma}{\kappa(\mu^2+(\beta+\theta+\omega+\gamma)\mu+(\omega+\gamma)\theta)}, \frac{-\beta(\mu^3+(\beta+\theta+\omega+\gamma)\mu^2+(\beta\omega-\kappa\Pi+\theta\omega+\theta\gamma)\mu-(\omega+\gamma)\kappa\Pi)}{\kappa(\mu^3+(\beta+\theta+2\omega+2\gamma)\mu^2+(\beta\omega+\beta\gamma+2\theta\omega+2\theta\gamma+\omega^2+2\omega\gamma)\mu+(\theta\omega^2+2\theta\omega\gamma+\theta\gamma^2))}\right) \quad (20)$$

with $\frac{\mu^2+(\beta+\theta+\omega+\gamma)\mu+(\beta+\theta)\omega+\theta\gamma}{\kappa(\mu+\omega+\gamma)} > 0, \frac{\mu^3-(\beta+\theta+\omega+\gamma)\mu^2+(\kappa\Pi-\theta\omega-\theta\gamma)\mu+\kappa\Pi\omega+\kappa\Pi\gamma}{\kappa(\mu^2+(\beta+\theta+\omega+\gamma)\mu+(\omega+\gamma)\theta)} > 0$ and $\frac{-\beta(\mu^3+(\beta+\theta+\omega+\gamma)\mu^2+(\beta\omega-\kappa\Pi+\theta\omega+\theta\gamma)\mu-(\omega+\gamma)\kappa\Pi)}{\kappa(\mu^3+(\beta+\theta+2\omega+2\gamma)\mu^2+(\beta\omega+\beta\gamma+2\theta\omega+2\theta\gamma+\omega^2+2\omega\gamma)\mu+(\theta\omega^2+2\theta\omega\gamma+\theta\gamma^2))} > 0$.

From the critical points (S_1, C_1, H_1) and (S^*, C^*, H^*), it can be explained that:

a. Critical point (S_1, C_1, H_1) will only exist in population S with a population size of $\frac{\Pi}{\mu}$. Meanwhile, population C will become extinct and implicated in the extinction of population H.

b. Critical point (S^*, C^*, H^*) shows the existence of the three populations S, C and H. The existence of each of these populations is influenced by parameters:
   1) $\frac{\mu^2+(\beta+\theta+\omega+\gamma)\mu+(\beta+\theta)\omega+\theta\gamma}{\kappa(\mu+\omega+\gamma)}$ for population S,
   2) $\frac{\mu^3-(\beta+\theta+\omega+\gamma)\mu^2+(\kappa\Pi-\theta\omega-\theta\gamma)\mu+\kappa\Pi\omega+\kappa\Pi\gamma}{\kappa(\mu^2+(\beta+\theta+\omega+\gamma)\mu+(\omega+\gamma)\theta)}$ for population C and
   3) $\frac{-\beta(\mu^3+(\beta+\theta+\omega+\gamma)\mu^2+(\beta\omega-\kappa\Pi+\theta\omega+\theta\gamma)\mu-(\omega+\gamma)\kappa\Pi)}{\kappa(\mu^3+(\beta+\theta+2\omega+2\gamma)\mu^2+(\beta\omega+\beta\gamma+2\theta\omega+2\theta\gamma+\omega^2+2\omega\gamma)\mu+(\theta\omega^2+2\theta\omega\gamma+\theta\gamma^2))}$ for population H.

After obtaining the critical point, it is necessary to test for stability. However, before conducting the stability test, it should be noted that the system of differential equations is nonlinear. It is necessary to linearize the system using Jacobian Matrix.

2. Linearization of Jacobian Matrix

Suppose equation (2) can be rewritten as:

$$\begin{gathered} f_1 = \Pi - \kappa CS + \omega H - \mu S \\ f_2 = \kappa CS - \beta C + \gamma H - \mu C - \theta C \\ f_3 = \beta C - \omega H - \gamma H - \mu H \end{gathered} \tag{21}$$

By finding the first derivative of f_1, f_2 and f_3 concerning S, C and H then Jacobian Matrix J is obtained

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial H} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial H} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial C} & \frac{\partial f_3}{\partial H} \end{pmatrix} = \begin{pmatrix} -\kappa C - \mu & -\kappa S & \omega \\ \kappa C & \kappa S - \beta - \mu - \theta & \gamma \\ 0 & \beta & -\omega - \gamma - \mu \end{pmatrix} \tag{22}$$

It can be seen that the Jacobian Matrix J is symmetrical 3 x 3 which is affected by the 3 populations in the system of equations.

3. Eigenvalues

From the Jacobian Matrix previously obtained, the eigenvalue λ of the Jacobian Matrix J. Eigenvalue λ can be found by $|\lambda I - J| = 0$ where I is the identity matrix corresponding to the Jacobian Matrix J and λ is scalar.

$$\left| \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} -\kappa C - \mu & -\kappa S & \omega \\ \kappa C & \kappa S - \beta - \mu - \theta & \gamma \\ 0 & \beta & -\omega - \gamma - \mu \end{pmatrix} \right| = 0 \tag{23}$$

Or it can be simplified to:

$$\begin{vmatrix} \lambda + \kappa C + \mu & \kappa S & -\omega \\ -\kappa C & \lambda - \kappa S + \beta + \mu + \theta & -\gamma \\ 0 & -\beta & \lambda + \omega + \gamma + \mu \end{vmatrix} = 0 \tag{24}$$

Suppose given a 3 x 3 matrix P with the following elements.

$$P = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \tag{25}$$

So, $\det(P)$ can be solve by:

$$|P| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - (ceg + bdi + afh) \tag{26}$$

Next, each critical point (S_1, C_1, H_1) and (S^*, C^*, H^*) into the equation (24). For (S_1, C_1, H_1), we get:

$$\begin{vmatrix} \lambda + \mu & \kappa \frac{\Pi}{\mu} & -\omega \\ 0 & \lambda - \kappa \frac{\Pi}{\mu} + \beta + \mu + \theta & -\gamma \\ 0 & -\beta & \lambda + \omega + \gamma + \mu \end{vmatrix} = 0 \tag{27}$$

So that the problem of equation (24) can be solved using equation (26) and obtained:

$$(\lambda + \mu)\left(\lambda - \kappa \frac{\Pi}{\mu} + \beta + \mu + \theta\right)(\lambda + \omega + \gamma + \mu) - \left((\lambda + \mu)(-\gamma)(-\beta)\right) = 0 \tag{28}$$

From equation (28), the eigenvalues for the critical points (S_1, C_1, H_1) as follows

1. $\lambda_{11} = -\mu$ and
2. $\lambda_{12} = \lambda_{13} = -\left(\mu + \frac{\beta+\theta+\omega+\gamma}{2} - \frac{\Pi\kappa}{2\mu}\right)\sqrt{O}$

where $O = (\beta^2 + 2\beta\theta - 2\beta\omega + 2\beta\gamma + \theta^2 - 2\theta\omega - 2\theta\gamma + \omega^2 + 2\omega\gamma + \gamma^2)\mu^2 + 2(\omega - \beta - \theta + \gamma)\kappa\Pi\mu + (\kappa^2\Pi^2)$.

In the same way, for the critical points (S^*, C^*, H^*) inputted into equation (24), it is obtained:

$$\lambda^3 + K\lambda^2 + L\lambda + M = 0 \tag{29}$$

with

$K = \beta - \kappa S + 3\mu + \kappa C + \omega + \gamma$

$L = (\kappa\beta + 2\kappa\mu + \kappa\theta + \omega\kappa + \gamma\kappa)C - (\kappa\mu + \omega\kappa + \gamma\kappa + \mu\kappa)S + \omega(\beta + \mu + \theta + \mu) + \gamma(\mu + \theta + \mu) + \mu(\beta + \mu + \beta + 2\mu + 2\theta) + \theta$

$M = (\omega\kappa\mu + \omega\kappa\theta + \gamma\kappa\mu + \gamma\kappa\theta + \mu\kappa\beta + \mu^2\kappa + \mu\kappa\theta)C - (\omega\kappa\mu + \gamma\kappa\mu + \kappa\mu^2)S + \mu(\mu^2 + \beta\mu + \mu\theta + \omega\beta + \omega\mu + \omega\theta + \gamma\mu + \gamma\theta)$

4. Stability Analysis

From the eigenvalues at the two critical points, a stability test will be conducted. A critical point is asymptotically stable if it satisfies $\lambda < 0, \forall\lambda \in \mathbb{R}$. For every variable and parameter > 0, the critical point (S_1, C_1, H_1) is asymptotically stable if:

a. $\lambda_{11} = -\mu < 0$ (already asymptotically stable)

b. $\lambda_{12} = \lambda_{13} = -\left(\mu + \frac{\beta+\theta+\omega+\gamma}{2} - \frac{\Pi\kappa}{2\mu}\right)\sqrt{O}$

where $O = (\beta^2 + 2\beta\theta - 2\beta\omega + 2\beta\gamma + \theta^2 - 2\theta\omega - 2\theta\gamma + \omega^2 + 2\omega\gamma + \gamma^2)\mu^2 + 2(\omega - \beta - \theta + \gamma)\kappa\Pi\mu + (\kappa^2\Pi^2)$. The stability condition is achieved if: $O > 0$ dan $\mu + \frac{\beta+\theta+\omega+\gamma}{2} > \frac{\Pi\kappa}{2\mu}$.

Furthermore, for the critical points (S^*, C^*, H^*) a will be asymptotically stable if in equation (29) the values of $K, L, M > 0$ with the conditions:

a. $\beta + 3\mu + \kappa C + \omega + \gamma > \kappa S$

b. $(\kappa\beta + 2\kappa\mu + \kappa\theta + \omega\kappa + \gamma\kappa)C + \omega(\beta + \mu + \theta + \mu) + \gamma(\mu + \theta + \mu) + \mu(\beta + \mu + \beta + 2\mu + 2\theta) + \theta > (\kappa\mu + \omega\kappa + \gamma\kappa + \mu\kappa)S$

c. $(\omega\kappa\mu + \omega\kappa\theta + \gamma\kappa\mu + \gamma\kappa\theta + \mu\kappa\beta + \mu^2\kappa + \mu\kappa\theta)C + \mu(\mu^2 + \beta\mu + \mu\theta + \omega\beta + \omega\mu + \omega\theta + \gamma\mu + \gamma\theta) > (\omega\kappa\mu + \gamma\kappa\mu + \kappa\mu^2)S$

Thus, it can be concluded that the critical points. (S_1, C_1, H_1) and (S^*, C^*, H^*) Are asymptotically stable conditionally.

Discussions

Based on the results above, it can be interpreted that the imposition of the death penalty on perpetrators of corruption can lead to 2 possibilities, including:

1. Corruptor population (C) Will become extinct and have implications for the extinction of the Honestly population (H) As well. Meanwhile, the Supectible population (S) will remain as much as the number of incoming populations divided by the natural death rate in the population S. This is supported by the results of the stability analysis which shows that the eigenvalues in this condition are conditionally asymptotically stable. In other words, it can be interpreted that if the death penalty sanction is imposed in Indonesia, it can exterminate the corrupt population. For this rule to be realized effectively and efficiently, the parameter aspects that affect it must also be optimized, including the number of individuals in the population, the Rate of spread of corruption from the population S to C, Rate of conversion to population H, Rate of natural death, and Rate of transition of population C to H.
2. Supectible population (S), Corruptor population (C), and Honestly population (H) Will still exist even though the death penalty has been imposed. This can happen because different parameter values affect these three populations so all three still exist. This possibility can occur because the results of the stability analysis show that the eigenvalues in this condition are conditionally asymptotically stable. Furthermore, it can be interpreted that the implementation of this regulation can be ineffective and inefficient if the government ignores the parameter aspects contained in the previous possibility.

The results of research on the Stability Analysis of Mathematical Models of Corrupt Behavior with the Death Penalty Sanction show that the death penalty can minimize and even destroy the number of corruptors if applied consistently and with high law enforcement. However, the effectiveness of the imposition of death penalty sanctions is an obstacle from the perspective of Corruption Crime Law and Human Rights as follows:

1. Corruption Crime Law

   Law of the Republic of Indonesia Number 31 of 1999 concerning Eradication of Corruption Crimes Chapter II Article 2 Paragraph (2) reads (Presiden RI, 1999):

"If the criminal act of corruption as intended in paragraph (1) is committed under certain circumstances, the death penalty can be dropped."

However, this regulation was changed by Law of the Republic of Indonesia Number 30 of 2002 concerning the Corruption Eradication Commission, where the regulation no longer contains the death penalty (Presiden RI, 2002). Subsequently, the Corruption Eradication Commission was enacted into law (Presiden RI, 2015). Furthermore, these regulations were amended again by Law of the Republic of Indonesia Number 19 of 2019 concerning the Second Amendment to Law Number 30 of 2002 concerning the Corruption Eradication Commission,. The death penalty has still been abolished. On the other hand, the Law of the Republic of Indonesia Number 1 of 2023 concerning the Criminal Code does not impose the death penalty on corruptors. Thus, it is clear that for the implementation of the death penalty, policy improvements in the criminal law of corruption need to be made.

2. Human Rights (HAM)

At the Plenary Session of Komnas HAM Number 07/SP/VI/2013 stated that Komnas HAM (Aswidah, 2016):

"Decided that the Komnas HAM Institutional position rejects its death penalty".

This is supported by the International Covenant on Civil and Political Rights, where the death penalty is applied with the following limitations:

"1. Death penalty cannot be applied except for the most serious crimes and by the laws in force at the time the crime took place, and 2. Anyone who is sentenced to death has the right to ask for pardon or commutation of sentence and can be given amnesty, pardon or commutation of sentence."

The statement is quite clear that Komnas HAM rejects the sanction of the death penalty except for the most serious crimes, such as terrorism, genocide, slavery, war crimes, etc.

Efforts to eradicate corruption are still a big task that is hampered by legal and human rights policies. Even though the death penalty is prohibited both at the national and international levels, several countries still impose the death penalty on corruptors even though international regulations prohibit it.

Table 7. Data on corruptors sentenced to death

No	Name of Corruptor	Country of Origin	Agency/ Department	Loss Value	Execution Time	Source
1	Li Jianping	China	Party Secretary	3 Billion Yuan	Dec, 17th 2024	CNN Indonesia, (2024)
2	Babak Zanjani	Iran	Oil Businessman	US$ 3 Billion	April 18th, 2024	Puspadini, (2024)
3	Jang Sung-Thaek	North Korea	Government Manager	Rp 1,4 Billion	Dec, 12th 2013	Mundari, (2021)
4	Truong My Lan	Vietnam	Property Businessman	US$ 12,5 Billon	April 11th, 2024	Yuliawati, (2024)

CONCLUSIONS, RECOMMENDATIONS, AND LIMITATIONS

Conclusions

Through the mathematical modeling approach using nonlinear differential equations, the imposition of death-penalty sanctions against corruptors can be analyzed by dividing the population into several categories, namely Susceptible (S), which refers to individuals who have the potential to commit corruption; Corruptor (C), which refers to individuals who are involved in corrupt practices; and Honest (H), which refers to individuals who repent and return to honest behavior. The stability analysis of the model indicates that without strict punishment, the number of corruptors within the government system tends to increase over time, particularly when the incentives for corruption are greater than the risks associated with punishment. In contrast, the implementation of the death penalty can significantly reduce the number of corruptors by creating a strong deterrent effect. Nevertheless, although theoretically the death penalty can serve as an effective solution to reducing corruption, its effectiveness in real-world application depends on legal considerations related to corruption and human rights. Therefore, the implementation of such a policy must take into account aspects of justice and transparency, as well as its broader social and economic impacts on the country.

Recommendation

This research provides policy implications that are mainly addressed to the government and law enforcement agencies, such as the Attorney General's Office, the Courts, the National Human Rights Commission, and the Corruption Eradication Commission, to consider the application of mathematical models in evaluating the effectiveness of the death penalty for corruptors. This model can be used to project the impact of punishment policies on the dynamics of the corrupt population to enable data-based decision-making. In addition, legislative bodies can use the results of this study to review regulations on corruption laws to optimize penalties that are not only repressive but also preventive. Academic institutions and researchers in the field of mathematical modeling can also use this study as a basis for developing more complex models, including economic, social, and political factors.

Limitations

The problem boundaries used as benchmarks in this research are defined through several key limitations. First, the model assumptions are based on the premise that each individual in the population behaves according to the laws of population dynamics, although this approach may not fully capture the complexity of human behavior in real-world situations in a detailed manner. Second, social and political factors are not considered in depth, as the research places greater emphasis on mathematical aspects rather than broader social, economic, and political influences, including possible public resistance or political intervention in the legal process. Third, the effectiveness of the model is highly dependent on the accuracy of the data employed, while empirical data regarding the impact of the death penalty on corruption levels remains very limited. Finally, the study does not analyze alternative punishments in depth, since it focuses primarily on the effectiveness of the death penalty in reducing corruption and therefore does not comprehensively compare it with other forms of punishment, such as life imprisonment or substantial financial penalties.

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